

SwiftCon China 2016

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从数学函数角度理解函数式编程

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//数学函数

$$f(x) = 1 + x$$

$$f(1) = 1 + 1 = 2$$

$$f(3) = 1 + 3 = 4$$

$$F(y) = 2y$$

$$F(1) = 2$$

$$F(3) = 6$$

//Swift 函数

```
func f(x: Int) -> Int {  
    return x + 1  
}
```

f(1)//2

f(3)//3

```
func F(y: Int) -> Int {  
    return 2 * y  
}
```

F(1)//2

F(3)//6

$F(y) = 2y$

$F(1.5) = 3.0$

```
func F(y: Int) -> Int {  
    return 2 * y  
}
```

```
F(1.5)  
//Cannot convert value of type 'Double'  
//to expected argument type 'Int'
```



```
func F(y: Double) -> Double {  
    return 2 * y  
}
```

```
F(1)//2.0
```

```
F(3)//6.0
```

```
F(1.5)//3.0
```

$$f(x) = 1 + x$$

$$F(y) = 2y$$

$$\begin{aligned} \text{令 } y &= f(x) \\ F(f(x)) &= 2 * f(x) \end{aligned}$$

$$\begin{aligned} g(x) &= F(f(x)) = 2f(x) \\ &= 2(x + 1) \end{aligned}$$

$$\begin{aligned} \text{令 } x &= 5 \\ g(5) &= F(f(5)) = 2 * 6 = 12 \end{aligned}$$

```
func f(x: Int) -> Int {  
    return x + 1  
}
```

```
func F(y: Int) -> Int {  
    return 2 * y  
}
```



```
func F(f: Int -> Int) -> Int -> Int {  
    func ret(x: Int) -> Int {  
        return 2 * f(x)  
    }  
    return ret  
}
```

```
let g = F(f)
```

```
g(5) // 12
```

高阶函数 (Higher-Order Functions)

```
func F(f: Int -> Int) -> Int -> Int {  
    func ret(x: Int) -> Int {  
        return 2 * f(x)  
    }  
    return ret  
}
```

闭包 (Closure)

"a closure is a record storing a function together with an environment"
-wiki

"闭包是指函数有自由独立的变量，定义在闭包中的函数可以“记忆”它创建时候的环境。”
-mozilla

为什么强调闭包？我们知道引入高阶函数是有其缘由和作用的，那么闭包到底是怎么出现的？

【3】 <https://developer.mozilla.org/zh-CN/docs/Web/JavaScript/Closures>

来具体的看一个闭包：

```
func F(f: Int -> Int) -> Int -> Int {  
    func ret(x: Int) -> Int {  
        return 2 * f(x)  
    }  
    return ret  
}
```



```
func F(f: Int -> Int) -> Int -> Int {  
    var fator: Int = 2  
    func ret(x: Int) -> Int {  
        return fator * f(x)  
    }  
    return ret  
}
```

闭包的概念及“闭包裹携其所处环境数据”的特点，正是为了闭包自己功能完整，不管闭包被赋值到哪里都能完整可用。

闭包、lambda、匿名函数的关系

Lambdas are a language construct (lambda is an anonymous function). closures are an implementation technique to implement first-class functions (whether anonymous or not)

Closures take one of three form:

Global functions are closures that have a name and do not capture any values.

Nested functions are closures that have a name and can capture values from their enclosing function.

Closure expressions are unnamed closures written in a lightweight syntax that can capture values from their surrounding context.

总结一下的话：lambdas就是匿名函数，一种匿名的形式或者表达式来表示函数。闭包跟匿名不匿名没什么关系，闭包更强调一种技术实现。但最终大家还是可以混着说。个人更倾向于swift定义，普通函数，lambdas，匿名函数都是闭包，闭包具体的实现了让各种函数是一等函数的能力。

swift语言在定义函数的时候，会用一个箭头 ->来表示函数的返回值，因为普通函数也是闭包，是一等函数，用->就显得非常统一合理。

//数学函数

$f(x, y) = x + 2y;$

$f(1, 2) = 5;$

令 $x = 1$, 我们会得到一个新的函数 $g(y)$:

$g(y) = f(1, y) = 1 + 2y;$

$g(2) = 1 + 4 = 5;$

//Swift 函数

```
func f(x: Int, _ y: Int) -> Int {  
    return x + 2 * y;  
}
```

$f(1,2)//5$



```
func f(x: Int) -> Int -> Int {  
    func ret(y: Int) -> Int {  
        return x + 2 * y  
    }  
    return ret  
}
```

$var g = f(1)$

$g(2)//5$

$f(1)(2)//5$

柯里化 (Currying)

在计算机科学中，柯里化 (Currying) 是把接受多个参数的函数变换成接受一个单一参数 (最初函数的第一个参数) 的函数，并且返回接受余下的参数且返回结果的新函数的技术。这个技术由 Christopher Strachey 以逻辑学家 Haskell Curry 命名的，尽管它是 Moses Schnfinkel 和 Gottlob Frege 发明的。

函数式编程 (functional programming)

什么是函数式编程，wiki页面的解释简单充分。

- `"In computer science, functional programming is a programming paradigm—a style of building the structure and elements of computer programs—that treats computation as the evaluation of mathematical functions and avoids changing-state and mutable data."`

这句话说函数式编程是一种编程范式，编程范式是一种编写程序的风格，函数式编程把函数写的像数学函数那样，编程计算就像函数运算，避免使用状态改变和可变数据类型。

- In functional code, the output value of a function depends only on the arguments that are input to the function, so calling a function f twice with the same value for an argument x will produce the same result $f(x)$ each time. Eliminating side effects, i.e. changes in state that do not depend on the function inputs, can make it much easier to understand and predict the behavior of a program, which is one of the key motivations for the development of functional programming.

这就话提到了进行函数式编程的一条主要动机：无副作用。数学中对函数的定义是从一个集合到另一个集合的映射，可以是多对一的映射但是不能是一对多。数学函数不会出现 $f(1)$ 即等于 a 又等于 b 的情况。正好对应无副作用的概念，或者是重入概念。无副作用和重入还有其它定义。这里强调的是：函数只要有相同的输入参数总会有相同的返回值，并且举了个例子，要消除函数里面状态改变语法，因为状态改变是不受函数参数控制的变化，这样的状态改变会引起副作用。

- Imperative programming does have functions—not in the mathematical sense—but in the sense of subroutines.

这一句说函数与函数式区别：指令式编程中的函数和数学函数没多大关系，指令式编程中的函数是一种subroutines（子程序）。Imperative programming 有的地方也叫Procedural programming（include C, Go, Fortran, Pascal, and BASIC）。以c语言来讲，c语言函数是一系列指令的打包，编译成汇编也是对应的一片指令，和数学函数概念相差很远。另外忽略运行时和虚拟机一些技术，c语言这类函数（subroutines、procedure）加载到内存后处于代码区（text segment、code segment），此区域是只读的，是不能在运行时的产生的。这类函数编译之后就是一系列的计算机指令和操作。可见与函数式编程中的一等函数差异是非常明显的。

最后一个小总结函数式编程的特征：

- 函数是一等函数，可以像其它数据类型出现在任何地方。
- 无副作用。
- 所有都是表达式，没有`for`，`switch`等状态控制。
- 使用函数组合来表达一连串的逻辑。

λ -calculus 、 Y combinator 和递归 (Recursion)

Motivating Example :

阶乘的数学递归定义: $0! = 1$, $n! = (n-1)! \times n$;

```
func factorial(n: Int) -> Int {  
    return n == 0 ? 1 : n * factorial(n - 1)  
}
```

```
factorial(3)//6
```

```
let factorial : Int -> Int = { (n: Int) -> Int in  
    return n == 0 ? 1 : n * factorial(n - 1)  
}
```

```
//Variable used within its own initial value  
//如何解决?
```


λ -calculus :

- Lambda calculus provides a theoretical framework for describing functions and their evaluation. Although it is a mathematical abstraction rather than a programming language, it forms the basis of almost all functional programming languages today. 【10】

Lambda calculus 由Alonzo Church提出，是一整套理论计算框架，一种数学抽象而不是具体的计算机语言。是函数式编程语言的理论基础。 λ 演算被证明是图灵等价。

看个最简单的Lambda calculus:

$\lambda x. x^2+2$

这个lambda表达式定义了一个匿名函数，这个函数的自变量是x。

对应的函数表示 $f(x) = x^2 + 2$; λx 表示x是自变量， x^2+2 是函数表达式 ;

$f(2) = 2^2 + 2 = 6$;

可以通用 λ 表达式表达 :

$(\lambda x. x^2+2)2$ //结果是6

Y Combinator就是下面这个lambda表达式：

$$Y : (\lambda y . (\lambda x . y(x x)) (\lambda x . y(x x)))$$

不动点：

$$x = f(x)$$

f是一个函数映射，当给f传入参数x，经过f的映射之后返回的值还是x。我们称x是函数f的不动点。

Y combinator的作用是可以找到函数的不动点。

$$Y : (\lambda y . (\lambda x . y(x x)) (\lambda x . y(x x))) \quad //Y$$

Combinator

$$YR = (\lambda x . R (x x)) (\lambda x . R (x x)) \quad //代入R, 进行计算$$
$$= R ((\lambda x . R (x x)) (\lambda x . R (x x)))$$
$$YR = R(YR) \quad //可见YR$$

是R的不动点！

正是Y combinator可得到一个函数的不动点的功效，让匿名函数递归成为可能。要实现匿名函数递归具体步骤如下：

Y Combinator:

```
func Y<A, B>(f: (A -> B) -> A -> B) -> A -> B {
    typealias RecursiveFunc = Any -> A -> B
    let r : RecursiveFunc = { (z: Any) in
        let w = z as! RecursiveFunc;
        return f {
            print(w.dynamicType)
            return w(w)($0)
        }
    }
    return r(r)
}
```

step 1:

```
typealias F = Int -> Int
var f:F -> F = {(f: F) -> F in
    var ret:Int -> Int = {(n: Int) -> Int in
        if (n == 0) {
            return 1;
        }
        return n * f(n - 1)
    }
    return ret;
}
```

step 2:

```
var factorial = Y(f);
factorial(3)//6
```

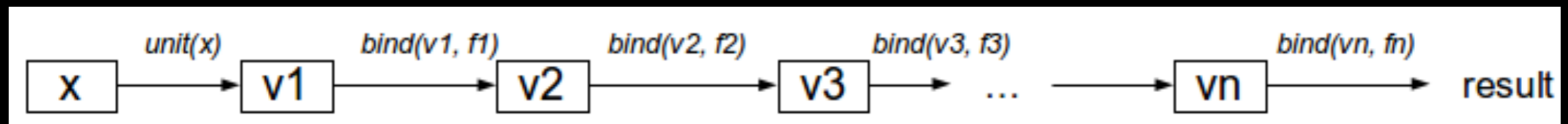
Swift & functional programming

1. `func Y<A, B>(f: (A -> B) -> A -> B) -> A -> B {}`

2. optional types:

“they are at the heart of many of Swift’s most powerful features.”

Monads



```

func payWaterBills(salary: Double) -> (Double, String) {
    if salary >= 100 {
        //pay water bills
        return (salary - 100, "")
    } else {
        return (salary, "payWaterBills error")
    }
}

func payElectricityBills(salary: Double) -> (Double, String) {
    if salary >= 300 {
        //pay electricity bills
        return (salary - 300, "")
    } else {
        return (salary, "payElectricityBills error")
    }
}

func payInternetBills(salary: Double) -> (Double, String) {
    if salary >= 500 {
        //pay internet bills
        return (salary - 500, "")
    } else {
        return (salary, "payInternetBills error")
    }
}

func tryPayBills2(salary: Double) -> (Double, String) {
    let salaryU = payWaterBills(salary)
    if salaryU.1 == "" {
        let salaryU = payElectricityBills(salaryU.0)
        if salaryU.1 == "" {
            let salaryU = payInternetBills(salaryU.0)
            return salaryU
        } else {
            return salaryU
        }
    } else {
        return salaryU
    }
}

print(tryPayBills2(30))
print(tryPayBills2(230))
print(tryPayBills2(630))
print(tryPayBills2(1000))

(30.0, "payWaterBills error")
(130.0, "payElectricityBills error")
(230.0, "payInternetBills error")
(100.0, "")

```

```
func unit(salary: Double) -> (Double, String) {
    return (salary, "")
}

func bind(salaryU:(Double, String), _ payBills: Double -> (Double, String)) -> (Double, String) {
    if salaryU.1 == "" {
        return payBills(salaryU.0)
    } else {
        return salaryU
    }
}

func tryPayBills1(salary: Double) -> (Double, String) {
    return bind(bind(bind(unit(salary), payWaterBills), payElectricityBills), payInternetBills)
}

print(tryPayBills1(30))
print(tryPayBills1(230))
print(tryPayBills1(630))
print(tryPayBills1(1000))

(30.0, "payWaterBills error")
(130.0, "payElectricityBills error")
(230.0, "payInternetBills error")
(100.0, "")
```

Optional Type:

```
var salary: Double? = Double("11")
```

Optional Binding: if let

```
if let salaryNum = salary {  
    print("my salary \(salaryNum)")  
} else {  
    print("something wrong.")  
}
```

Optional Chaining:

```
if let buildingIdentifier = john.residence?.address?.buildingIdentifier() {  
    print("John's building identifier is \(buildingIdentifier).")  
}  
// Prints "John's building identifier is The Larches."
```

nil:

Swift's nil is not the same as nil in Objective-C. In Objective-C, nil is a pointer to a nonexistent object. In Swift, nil is not a pointer—it is the absence of a value of a certain type. Optionals of any type can be set to nil, not just object types.

多个角度理解 : Optional / if let


```
var x: String???
```

```
//Monads, enumeration
```

value	nil	nil	nil
-------	-----	-----	-----

```
x = nil;
x = Optional.None;
x = Optional.Some(Optional.None);
x = Optional.Some(Optional.Some(Optional.None));
x = Optional.Some(Optional.Some(Optional.Some("value")));
x = "value";

if let s = x {
    print(" \(\x)")
    print(" \(\s)")
} else {
    print("there is no s !")
}

if x != nil {
    let s = x!
    print(" \(\x)")
    print(" \(\s)")
} else {
    print("there is no s !")
}
```

Thank You