Cosine Transform and Its Applications

Instructor

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Outline

- ❖Discrete Cosine Transform (DCT)
- ❖Its Applications

1-D DISCRETE COSINE TRANSFORM
DCT

$$C(u) = a(u)\sum_{x=0}^{N-1} f(x)\cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

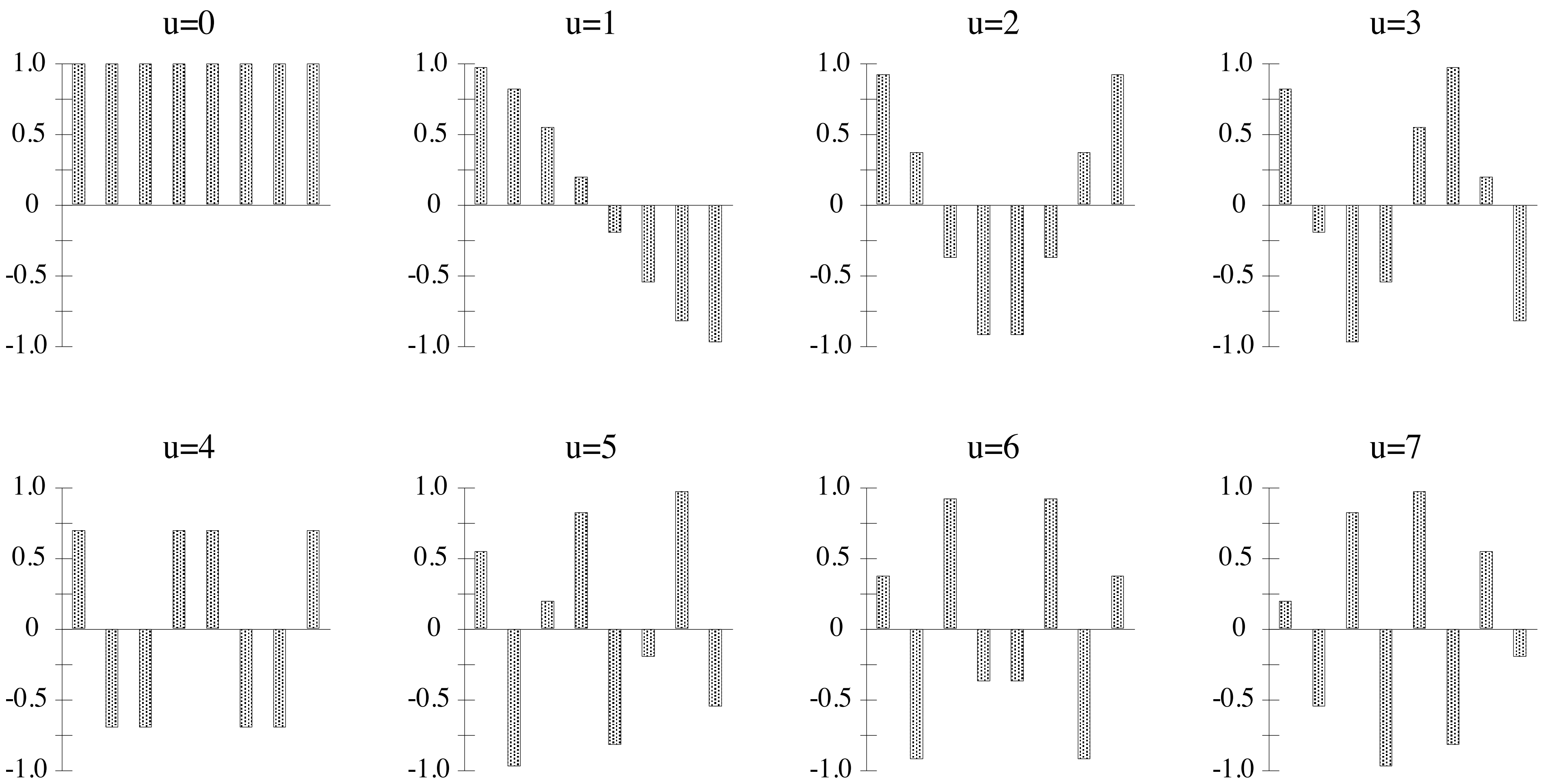
$$u = 0,1,\ldots,N-1$$

$$a(u) = \begin{cases} \sqrt{\dfrac{1}{N}} & u = 0 \\ \sqrt{\dfrac{2}{N}} & u = 1,\ldots,N-1 \end{cases}$$

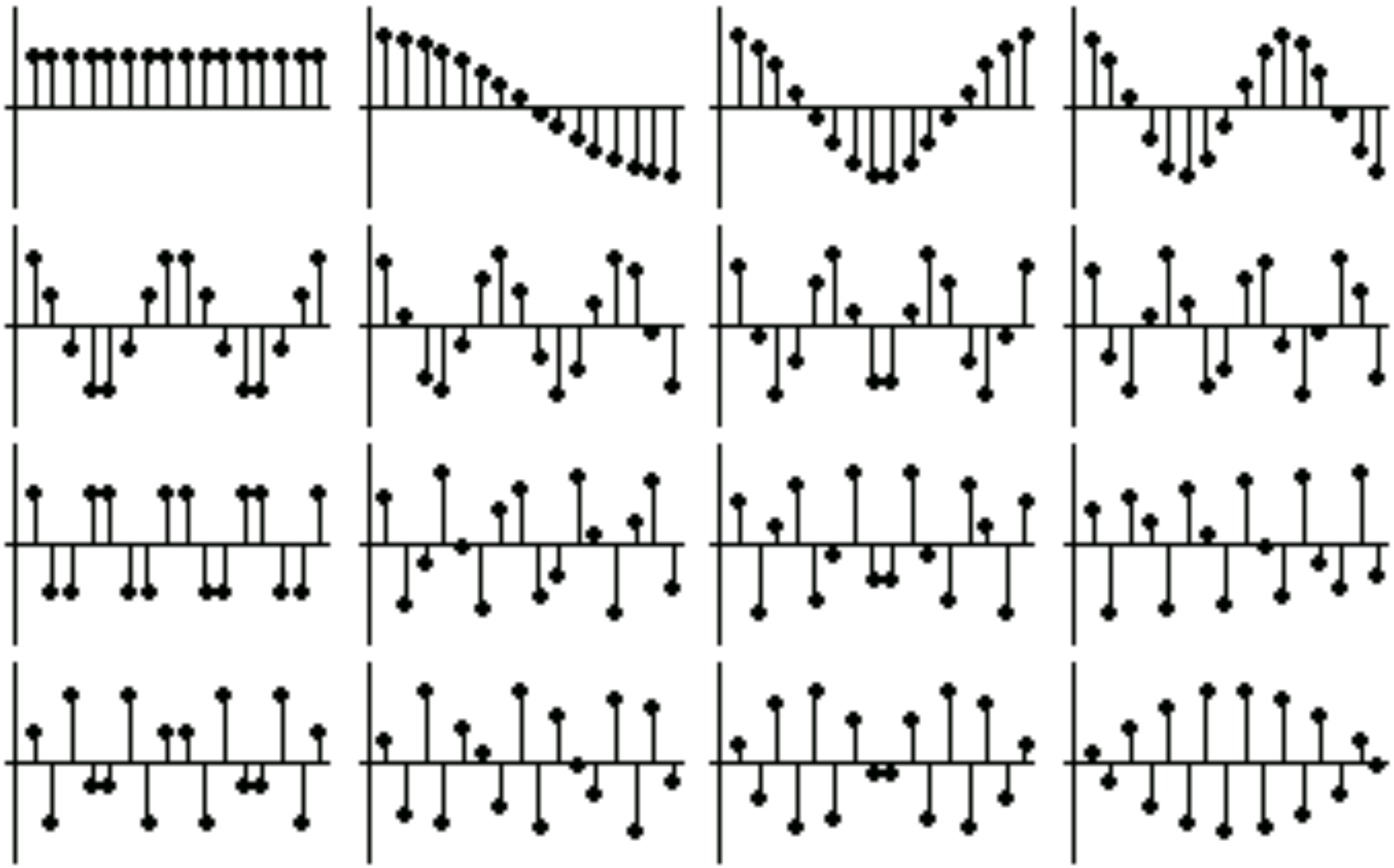
1-D INVERSE DISCRETE COSINE TRANSFORM
IDCT

$$f(x) = \sum_{u=0}^{N-1} a(u)C(u)\cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

1-D Basis Functions $N=8$

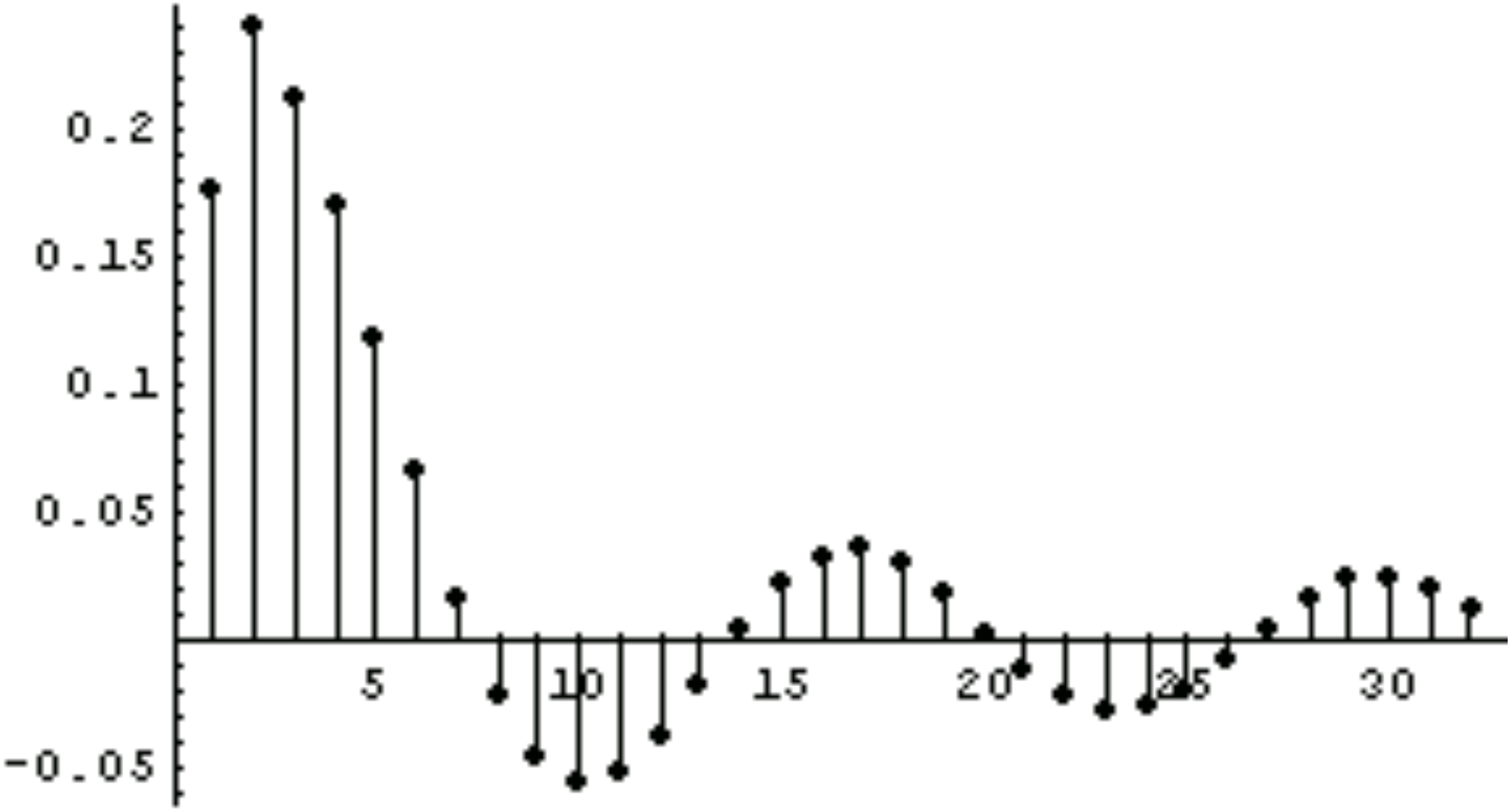


1-D Basis Functions N=16

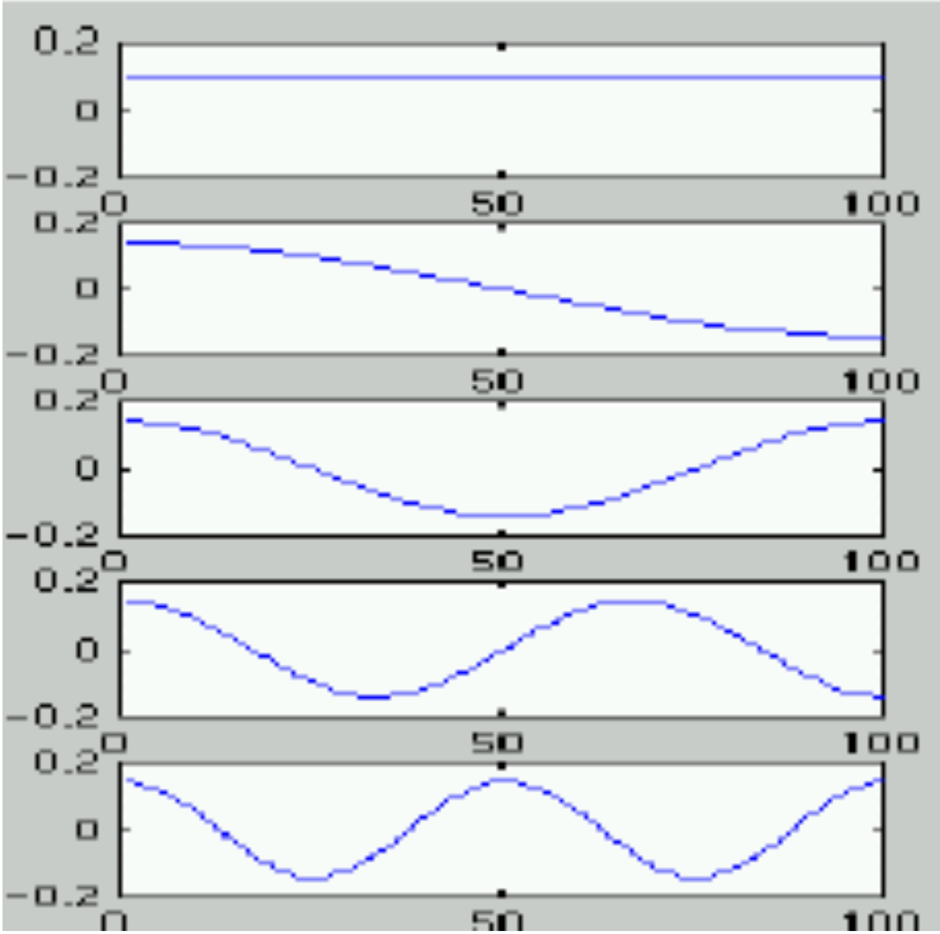


Example: 1D signal

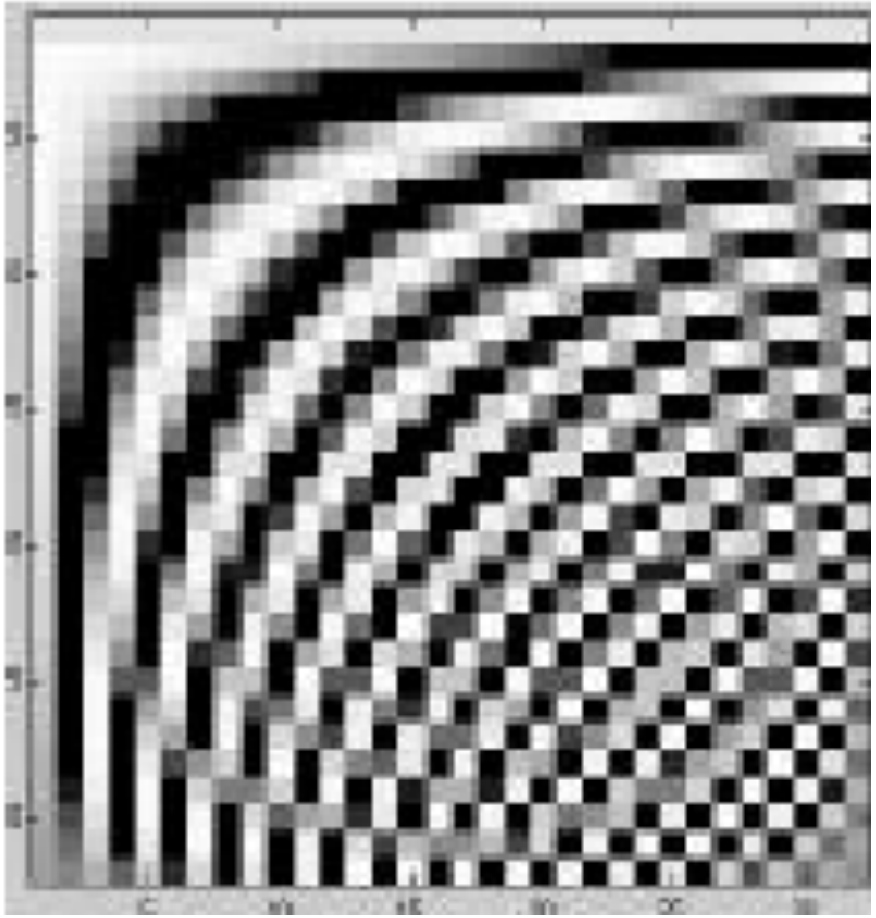
$$x[n] = \begin{cases} \frac{1}{5}, & \text{for } 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$



- First 5 vectors:



- Image of full 32x32:



2-D DISCRETE COSINE TRANSFORM
DCT

$$C(u,v) = a(u)a(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{(2x+1)u\pi}{2N}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

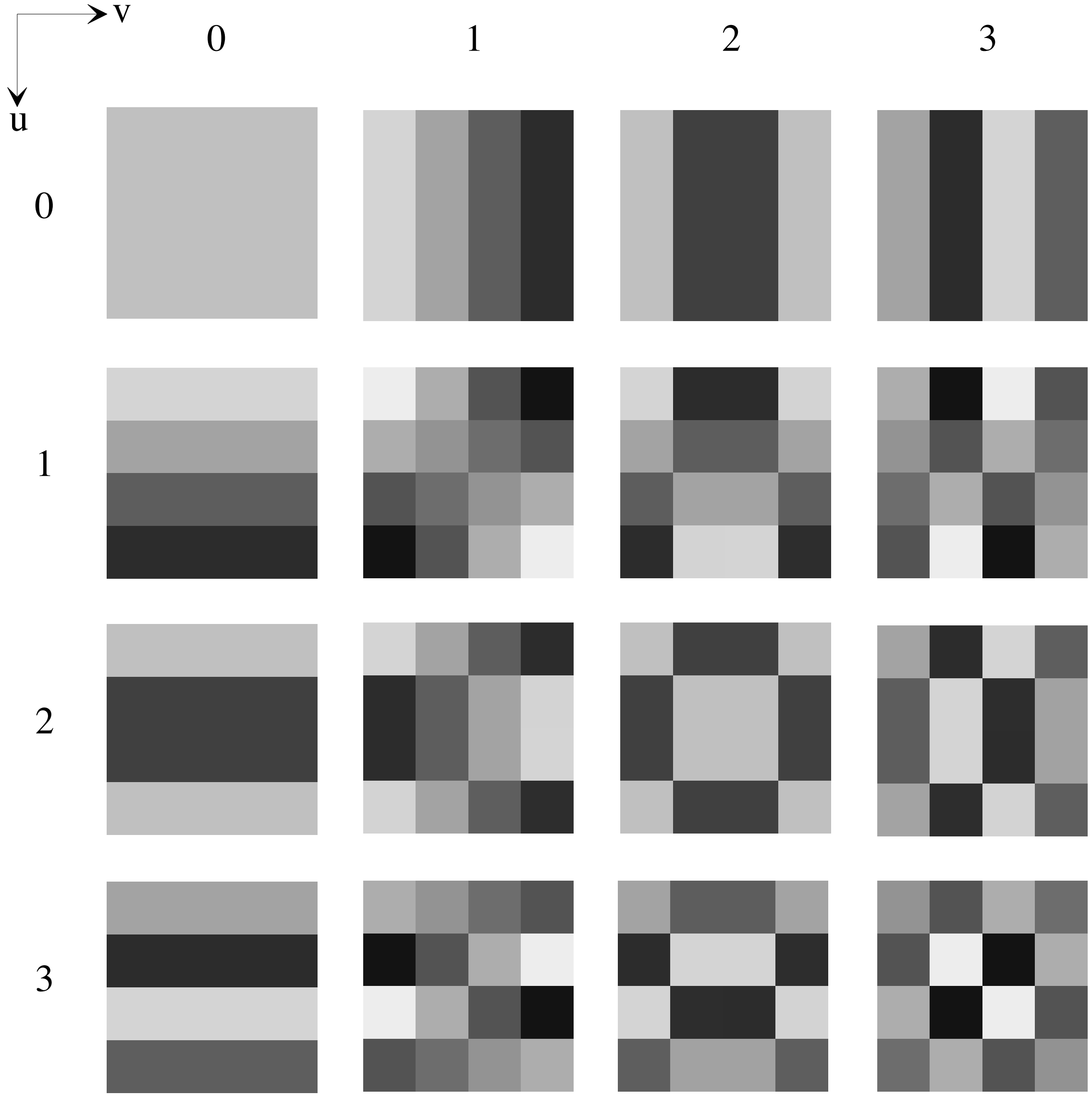
$$f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} a(u)a(v)C(u,v)\cos\left[\frac{(2x+1)u\pi}{2N}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

$$u,v = 0,1,\ldots,N-1$$

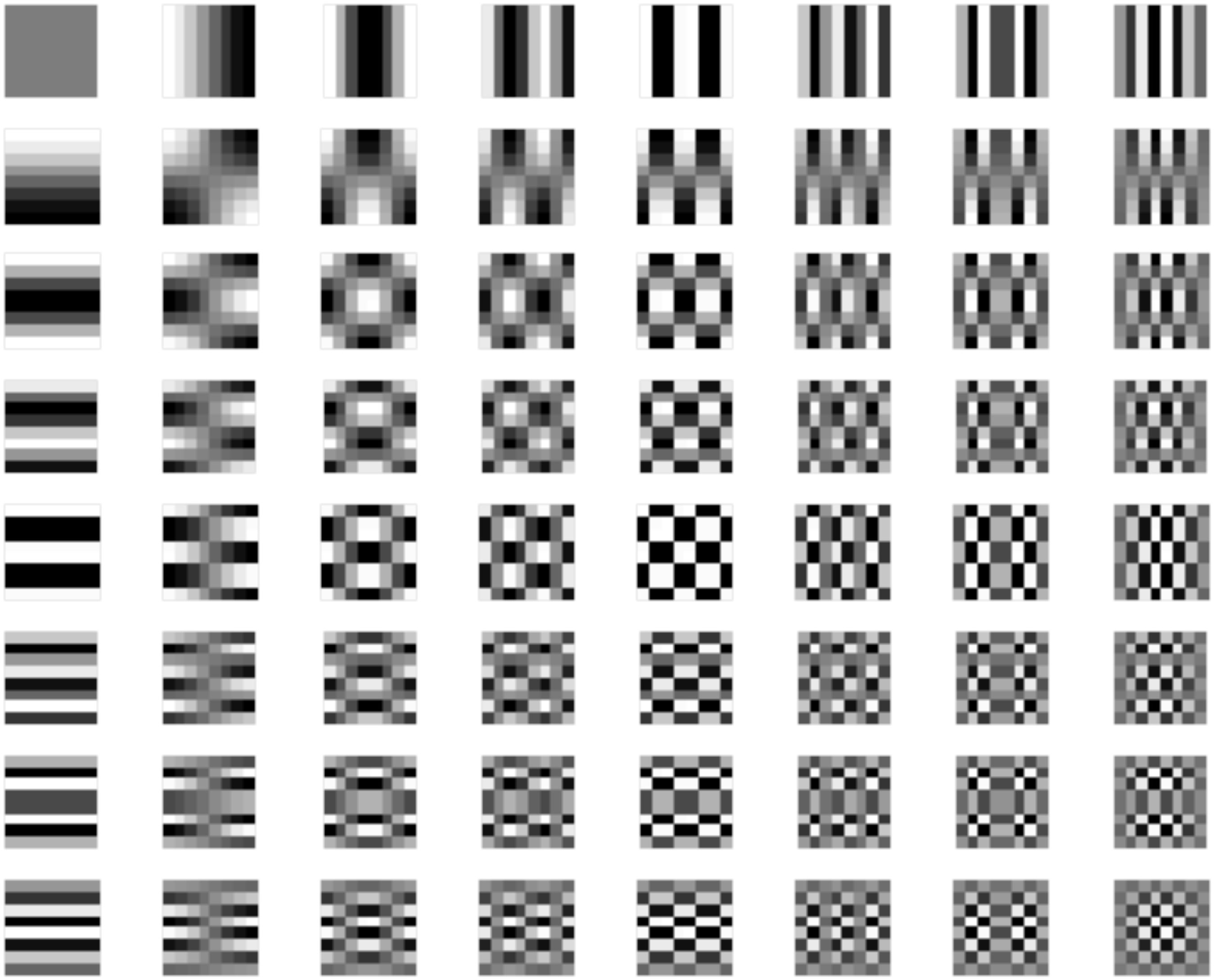
ADVANTAGES

- Notice that the DCT is a real transform.
- The DCT has excellent energy compaction properties.
- There are fast algorithms to compute the DCT similar to the FFT.

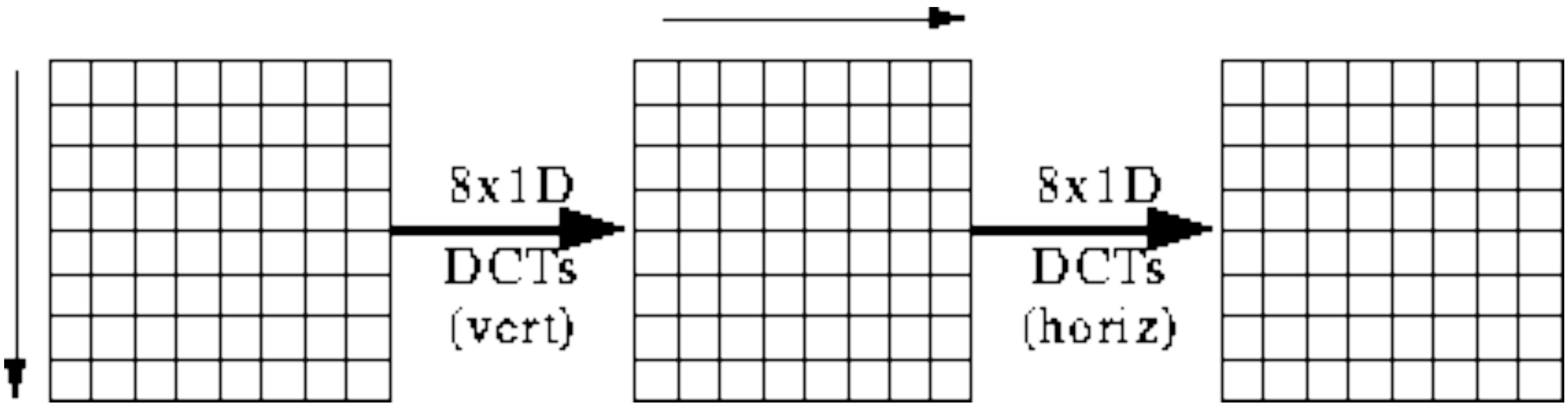
2-D Basis Functions *N*=4



2-D Basis Functions *N*=8

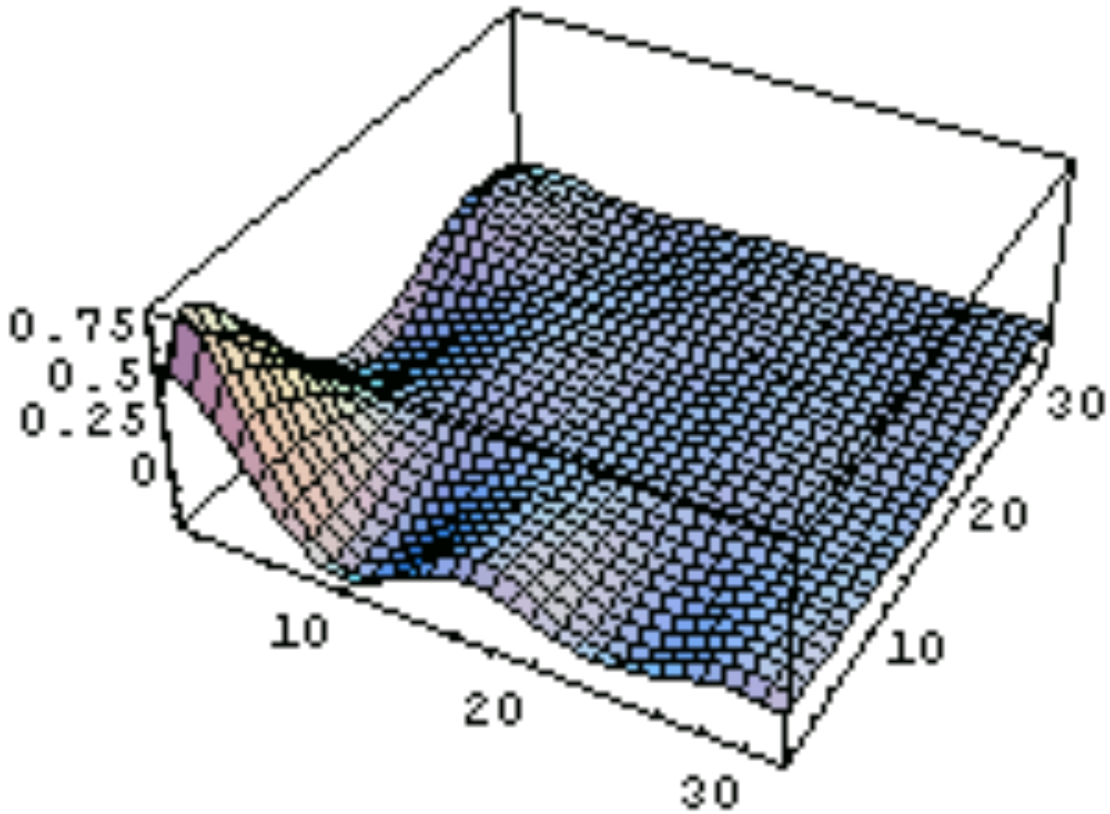


Separable

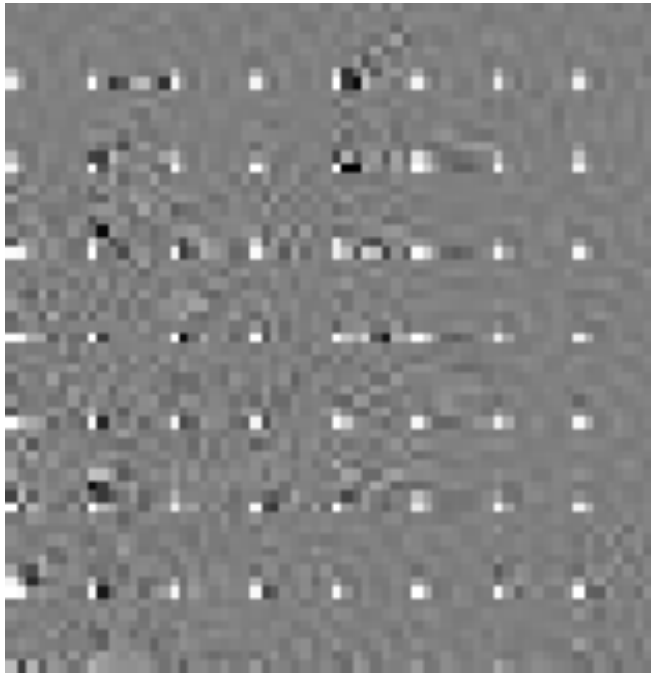


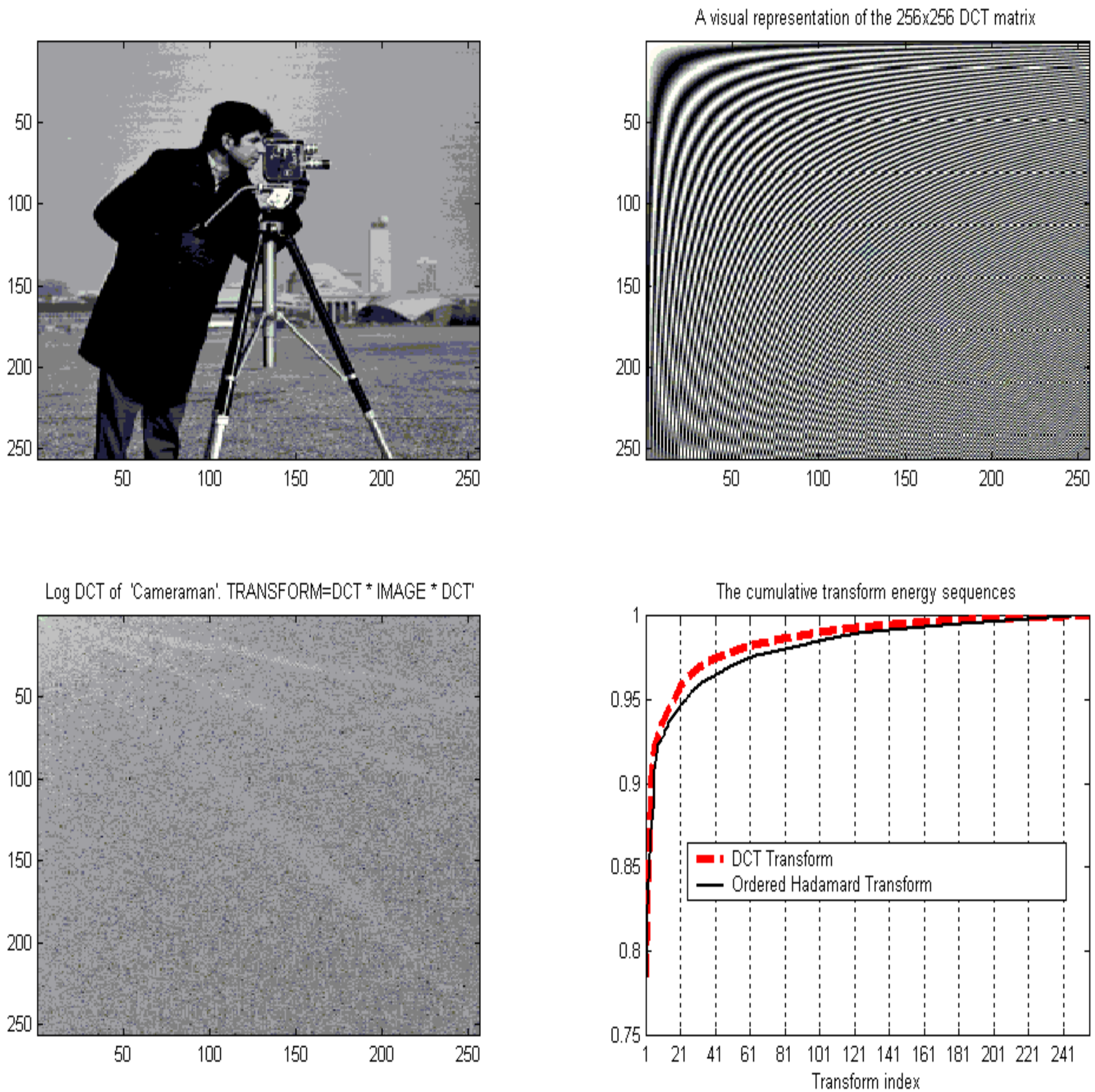
Example: 2D signal

$$x[n_1, n_2] = \begin{cases} 1, & 0 \le n_1 \le 2, \quad 0 \le n_2 \le 4 \\ 0, & \text{otherwise} \end{cases}$$



8x8 Block DCT




A visual representation of the 256x256 DCT matrix
Log DCT of 'Cameraman'. TRANSFORM=DCT * IMAGE * DCT'
The cumulative transform energy sequences
DCT Transform
Ordered Hadamard Transform
Transform index
50
100
150
200
250
1
0.95
0.9
0.85
0.8
0.75
1 21 41 61 81 101 121 141 161 181 201 221 241

Example: Energy Compaction

- Original Lena image

- 2D DCT

Relation between DCT and DFT

❖ Define

$$g(x) = f(x) + f(2N-1-x)$$

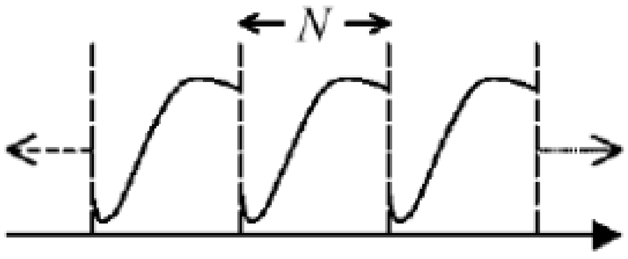
$$= \begin{cases} f(x), & 0 \le x \le N-1 \\ f(2N-1-x), & N \le x \le 2N-1 \end{cases}$$

$N-\text{point}$		$2N-\text{point}$	DFT	$2N-\text{point}$		$N-\text{point}$
$f(x)$	$\rightarrow$	$g(x)$	$\rightarrow$	$G(u)$	$\rightarrow$	$C_f(u)$

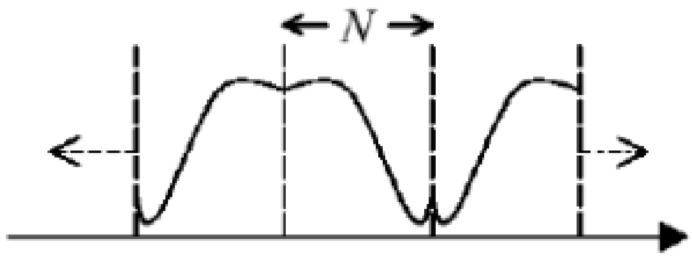
From DFT to DCT (Cont.)

DCT has a higher compression ration than DFT

- DCT avoids the generation of spurious spectral components

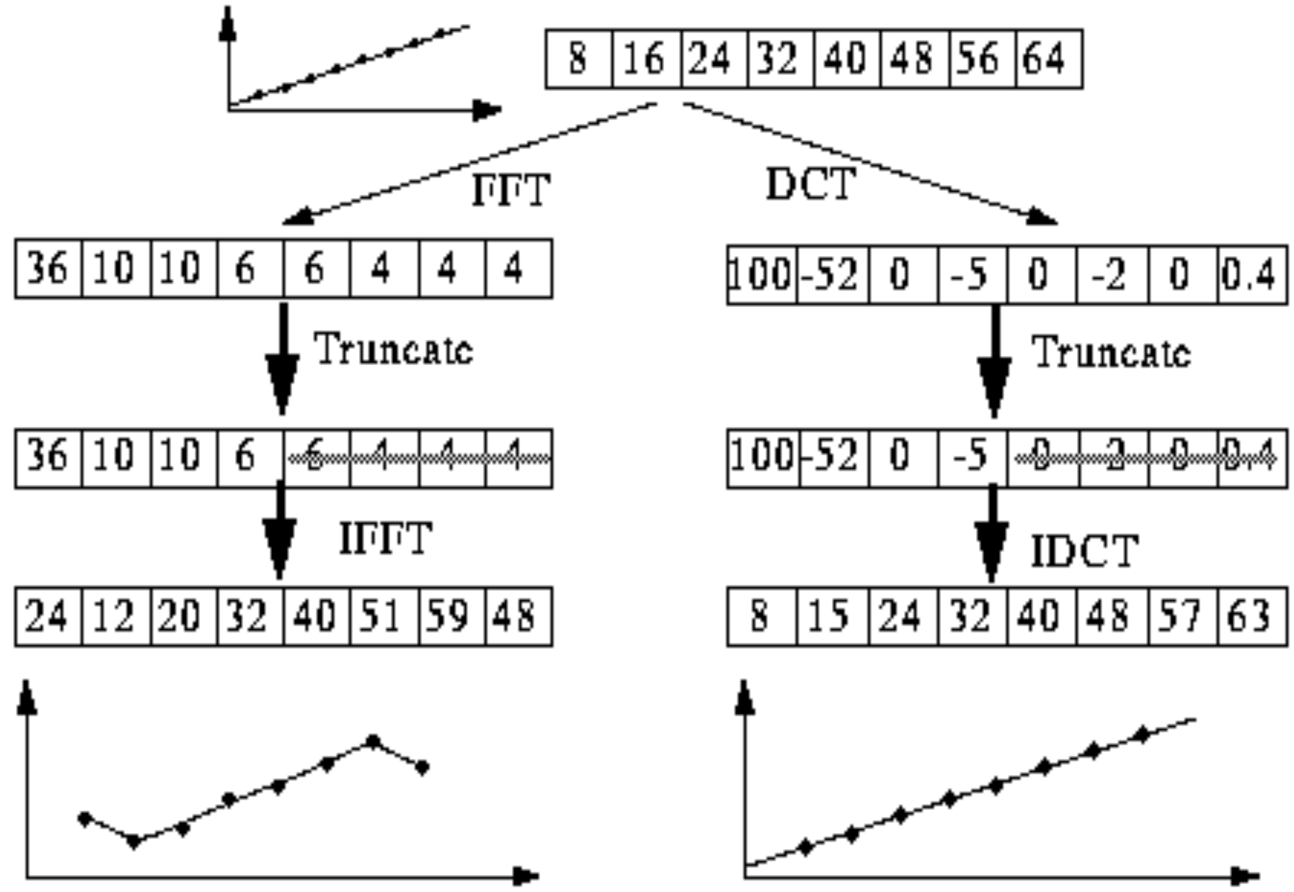


DFT periodicity



DCT periodicity

DCT vs. DFT



DCT/FFT Comparison

Transform (DCT)